Optimal Design of Antisymmetric Laminated Composite Plates

N.G.R. Iyengar*
University of Arizona, Tucson, Arizona
and
S.P. Joshi†
Purdue University, West Lafayette, Indiana

Abstract

IN this investigation, the minimum weight design of a laminated fiber-reinforced composite plate subjected to in-plane and transverse loading is attempted. Restrictions are imposed on the buckling load and transverse deflection. The fiber orientation and thickness of each ply are treated as design variables. Optimization studies are carried out by using an unconstrained minimization technique. Numerical results have been obtained for antisymmetric angle-ply laminates treating number of plies as a parameter. Some of the observations are that, with preassigned fiber orientation, the optimum weight design results in a unique thickness distribution of the plies and that the stability constraint is active at low aspect ratios, while the deflection constraint is active at large aspect ratios.

Contents

In the last two decades, the use of fiber-reinforced composites as load-carrying members has increased considerably. This increase is due to the fact that the designer can take advantage of the anisotropic properties of these materials. Advanced composite materials offer a significant potential for reducing the weight of aerospace structural components. One of the important parameters that govern the design of aerospace vehicles is the structural weight. One can get a minimum structural weight design by employing the optimization techniques.

From the available literature,[1] it appears that not all the design variables have been considered in arriving at an optimum design. Therefore, the present work attempts to study the optimum design of laminated composite plates by taking into account all the possible design variables. In a recent paper,[2] the authors presented their studies on the minimum weight design of symmetric laminated composite plates subjected to in-plane and transverse loading. In the investigation reported here, an attempt is made to obtain a minimum weight design for antisymmetric angle-ply laminates incorporating thickness and fiber orientation of each ply as design variables, while treating the number of plies as a design parameter. Numerical results are presented for boron/epoxy laminates for varying aspect ratio and number of plies.

Optimization Formulation

It is assumed that the material is the same for all the laminae. The objective function for weight, therefore, reduces to

$$F = \min \sum_{i=1}^{N/2} t_i \tag{1}$$

where $N/2$ is the number of plies in the laminate and t_i the thickness of the ith ply.

The behavior constraints are 1) the deflection W of the plate for a given transverse loading is less than the prescribed value W^* and 2) the buckling load N_x of the plate is greater than the prescribed value N_x^*. The side constraints are the lower and upper bounds on thickness of each ply.

The optimization problem as stated above is termed one of a constrained optimization. This constrained problem can be transformed into a series of unconstrained problems by introducing a penalty term associated with the constraints. The problem then reduces to

$$P(r,t_i) = F(t_i) - r\sum_{j=1}^{m} f_j(\alpha_i,t_i) \tag{2a}$$

where m is the total number of constraints and $f_j(\alpha_i,t_i)$ is defined as

$$f_j(\alpha_i,t_i) = \ln[g_j(\alpha_i,t_i)] \qquad \text{if} \qquad g_j(\alpha_i,t_i) \geq 0 \tag{2b}$$

The term $rf_j(\alpha_i,t_i)$ represents the penalty associated with the jth constraint in an interior penalty function in the sense that it is defined only if the design vector lies in the feasible domain. Fletcher-Powell's variable metric method has been used for each unidirectional search in the Sequential Unconstrained Minimization Technique (SUMT) algorithm.[3]

Analyses for Buckling and Deflection

Reference 4 gives the governing equations for an antisymmetric angle-ply laminate. The displacement functions are chosen to satisfy the simply supported boundary conditions of the plate.

For the in-plane, biaxial compressive loading N_x and N_y, the resulting buckling load equation is

$$N_x \frac{1}{\pi^2 a^2 [m^2 + Kn^2 (a/b)^2]} \left[T_{33} \frac{2T_{12}T_{13}T_{23} - T_{22}T_{13}^2 - T_{11}T_{23}^2}{T_{11}T_{22}T_{12}^2} \right] \tag{3}$$

where $K = N_y/N_x$ and a and b are the dimensions of the plate along the x and y directions, respectively. The T_{12} in Eq. (3)

Table 1 Variation of optimum weight with fiber orientation[a]

Fiber orientation α, deg	Optimum thickness t_1	t_2	Optimum weight, $W/\rho A$	t_2/t_2
± 45	0.4420	1.0666	3.0172	0.414
± 30	0.4103	0.9885	2.7976	0.415
± 10	0.3928	0.9488	2.6832	0.414

[a] $N = 4$, aspect ratio $= 5$, and $k = 0.5$.

Presented as Paper 85-0796 at the AIAA 26th Structures, Structural Dynamics and Materials Conference, Orlando, FL, April 15-17, 1985; received Oct. 1, 1985; synoptic received Dec. 2, 1985. Full paper available from AIAA Library, 555 W. 57th St., New York, NY 10019. Price: Microfiche, $4.00; hard copy $9.00. **Remittance must accompany order.**

*Visiting Professor, Aerospace and Mechanical Engineering Department.

†Visiting Professor, Department of Aeronautics and Astronautics (presently, Assistant Professor, Aerospace and Mechanical Engineering Department, University of Arizona, Tucson).

are defined as

$$T_{11} = A_{11}m^2\pi^2 + A_{66}n^2\pi^2(a/b)^2$$

$$T_{12} = (A_{12} + A_{66})mn\pi^2(a/b)^2$$

$$T_{13} = -[3B_{16}m^2\pi^2 + B_{26}n^2\pi^2(a/b)^2]n\pi(a/b)$$

$$T_{22} = A_{66}m^2\pi^2 + A_{22}n^2\pi^2(a/b)^2$$

$$T_{23} = -[B_{16}m^2\pi^2 + 3B_{26}n^2\pi^2(a/b)^2]m\pi$$

$$T_{33} = D_{11}m^4\pi^4 + 2(D_{12} + 2D_{66})m^2n^2\pi^4(a/b)^2 + D_{22}n^4\pi^4(a/b)^4 \tag{4}$$

For a plate subjected to a uniform transverse load q_0 per unit area, the displacement coefficients U_{mn}, V_{mn}, and W_{mn} are

$$U_{mn} = \frac{q_{mn}}{a^5\bar{D}}(T_{22}T_{13} - T_{12}T_{23})$$

$$V_{mn} = \frac{q_{mn}}{a^5\bar{D}}(T_{11}T_{23} - T_{13}T_{12})$$

$$W_{mn} = \frac{q_{mn}}{a^4\bar{D}}(T_{22}T_{11} - T_{12}^2) \tag{5}$$

in which the transverse loading $q(x,y)$ is expressed in terms of the Fourier series as

$$q \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{6}$$

where

$$q_{mn} = \frac{16q_0}{\pi^2}\cdot\frac{1}{mn} \quad \text{and} \quad m,n = 1,3,5,\ldots \tag{7}$$

$\bar{D}$ in Eqs. (5) is given by

$$\bar{D} = \frac{T_{11}T_{22} - T_{12}^2}{a^8}\left[T_{33} + \frac{2T_{12}T_{13}T_{23} - T_{22}T_{13}^2 - T_{11}T_{23}^2}{T_{11}T_{22} - T_{12}^2}\right] \tag{8}$$

Numerical Computations

Numerical studies have been carried out for rectangular composite plates with number of plies $N/2$ and aspect ratio a/b as parameters for boron/epoxy composites whose material properties are as follows:

$$E_L = 2.11 \times 10^4 \text{ kg/mm}^2 \qquad E_T = 2.11 \times 10^3 \text{ kg/mm}^2$$

$$\nu_{LT} = 0.3 \qquad G_{LT} = 7.03 \times 10^2 \text{ kg/mm}^2$$

The values for the transverse load, behavior, and side constraints are taken as

$$q_0 = 0.0005 \text{ kg/mm}^2, \quad W^* = 2.00 \text{ mm}, \quad N_x^* = 5.00 \text{ kg/mm}$$

$$t_\ell = 0.125 \text{ mm}, \quad t_u = 2.5 \text{ mm}$$

Results and Discussion

The results for a four-ply antisymmetric laminate of aspect ratio 0.5 and in-plane loading ratio $K = 0.5$ are shown in Table 1. Here, the fiber orientations are preassigned and only the ply thickness is treated as a design variable. It is observed that a minimum weight design is attained when the fibers are approximately ± 10 deg. Further, at this design point, the ply thickness is 0.414. This thickness ratio is the same as reported by Sharma et al.[5]

For a six-ply laminate, Sharma et al.[5] have shown that coupling effects can be completely eliminated if the thickness of plies satisfies the following equation:

$$\frac{t_2}{t_3} = \left(\frac{t_1}{t_3} - 1\right) \pm \left[2\left(\frac{t_1}{t^3}\right)^2 + 2\right]^{1/2}$$

The thickness ratio obtained from the analysis satisfies the above equation exactly. The increase in the number of plies makes a significant change in the ply thickness, but the optimum weight remains constant.

Results have also been obtained for a four-ply laminate where thickness and fiber orientation of each ply are treated as design variables. It is found that the optimum weight increases with increasing aspect ratio a/b and biaxial loading ratio K. The thickness ratio decreases with increasing biaxial loading ratio. Further, the active constraint at the optimum weight depends on the aspect ratio. This behavior is possible since, from Eqs. (3) and (5), it is seen that at lower aspect ratios, the buckling constraint is active; while, at higher aspect ratios, the constraint on deflection is active. The optimum weight does not depend on the biaxial loading ratio if the constraint on deflection is active.

Conclusions

On the basis of the investigations on the antisymmetric angle-ply composite laminate, the following conclusions can be drawn:

1) A specific thickness distribution appears to be valid for optimum weight design of laminated plates with preassigned fiber orientation.

2) The weight per unit area of the plate increases with increasing aspect ratio.

3) The stability constraint is active at low aspect ratios, while the deflection constraint is active at high aspect ratios.

4) When both thickness and fiber orientation of plies are treated as design variables, the thickness ratio and fiber orientation reach an optimum value.

References

[1] Iyengar, N.G.R. and Joshi, S.P., "Optimal Design of Antisymmetric Laminated Composite Plates," AIAA Paper 85-0796, 1985.

[2] Joshi, S.P. and Iyengar, N.G.R., "Studies on Optimization of Laminated Composite Plates," *Proceedings of 13th Congress of International Council on Aeronautical Sciences,* Seattle, 1982, pp. 607-614.

[3] Iyengar, N.G.R. and Gupta, S.K., *Programming Methods in Structural Design,* Edward Arnold, Ltd, London, 1981.

[4] Jones, R.M., *Mechanics of Composite Materials*, Scripta Book Co., Washington, D.C., 1975.

[5] Sharma, S., Iyengar, N.G.R., and Murthy, P.N., "Some Comments on Coupling of Angle-Ply Laminates," *Journal of Structural Mechanics,* Vol. 7, 1979, pp. 473-482.